ALLOWANCE FOR MUTUAL EFFECT OF "HOT SPOTS" IN SOLUTION OF PROBLEMS OF SITE THERMAL EXPLOSION

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Local effect of intensification of substance heating depending on the distance between "hot spots" is studied within the framework of a site thermal explosion in the case of a zeroth-order reaction. A combination of the values of the Arrhenius number Ar, Frank-Kamenetskii criterion Fk, distance between "hot spots" a, and their width w at which two nearest points give a nonlinear effect of local intensification of the total heating of a substance is found. The method is based on the use of an approximate analytical solution of the corresponding Cauchy problem which is presented in the form of asymptotic expansion in terms of Poincaré.

Although classical thermal-explosion theory presumes the uniformity of the field of temperatures and concentrations at the initial instant of time, nevertheless in a number of works [1-3] the effect of initial nonuniformity – "hot spots" – on the development of a site thermal explosion was studied theoretically. At the same time, it was stated that the "issue of the development of reaction sites in the case of an arbitrary initial disturbance cannot be solved by numerical methods" [2]. We emphasize that the problem of ignition of a system of hot sites has both theoretical and practical value, since condensed particles, formed in the combustion of pyrotechnic compositions and gunpowder, while getting onto the surface of a charge and penetrating into the monolith of a solid fuel, form ignition sites scattered over the surface [4].

This work deals with the mutual effect of a periodic system of heating sites, each of which is modeled by the initial Gaussian temperature distribution. It is obvious that by virtue of the assumption of periodicity of this system, it suffices to study the mutual effect of only two sites of initiation. This work continues a parametric analytical study of modes of a site thermal explosion started in [5], where the principles of using of a "geometrical optical" asymptotic method for an analytical parametric investigation of the site modes of thermal explosion in the case of one site of initiation are formulated; comparison with the results obtained by other authors is made and the conditions, under which the effect of "traveling thermal waves" is realized, are revealed.

We set ourselves the task of answering the question: under what conditions (i.e., for what values of the Arrhenius number Ar, width of "hot spots" w, and distance a between them at fixed values of the Frank-Kamenetskii criterion Fr) are "traveling thermal waves" imposed on each other, thus producing the effect of intensification?

A mathematical formulation of the problem of a site thermal explosion (without regard for the burnout of a substance – a zeroth-order reaction) in dimensionless variables suggested by Frank-Kamenetskii is presented in [5] by formulas (1)-(3).

The smallness of the parameter $\varepsilon = \mathrm{Fk}^{-1}$ is a special property of a site thermal explosion in condensed media [1-3]. This makes it possible to write the solution $\Theta(\xi, \tau)$ of the Cauchy problem in the form of relation (4) from [5], analytical expressions for the coefficients of expansion of which have the form (5)-(9) from [5]. The same work gives the details of derivation of formulas (4)-(9).

The initial distribution $\Theta^0(\xi)$ of the dimensionless heating $\Theta(\xi, \tau)$ was assigned by two Gaussian curves; in the adopted dimensionless system of coordinates, this corresponds to an analytical expression

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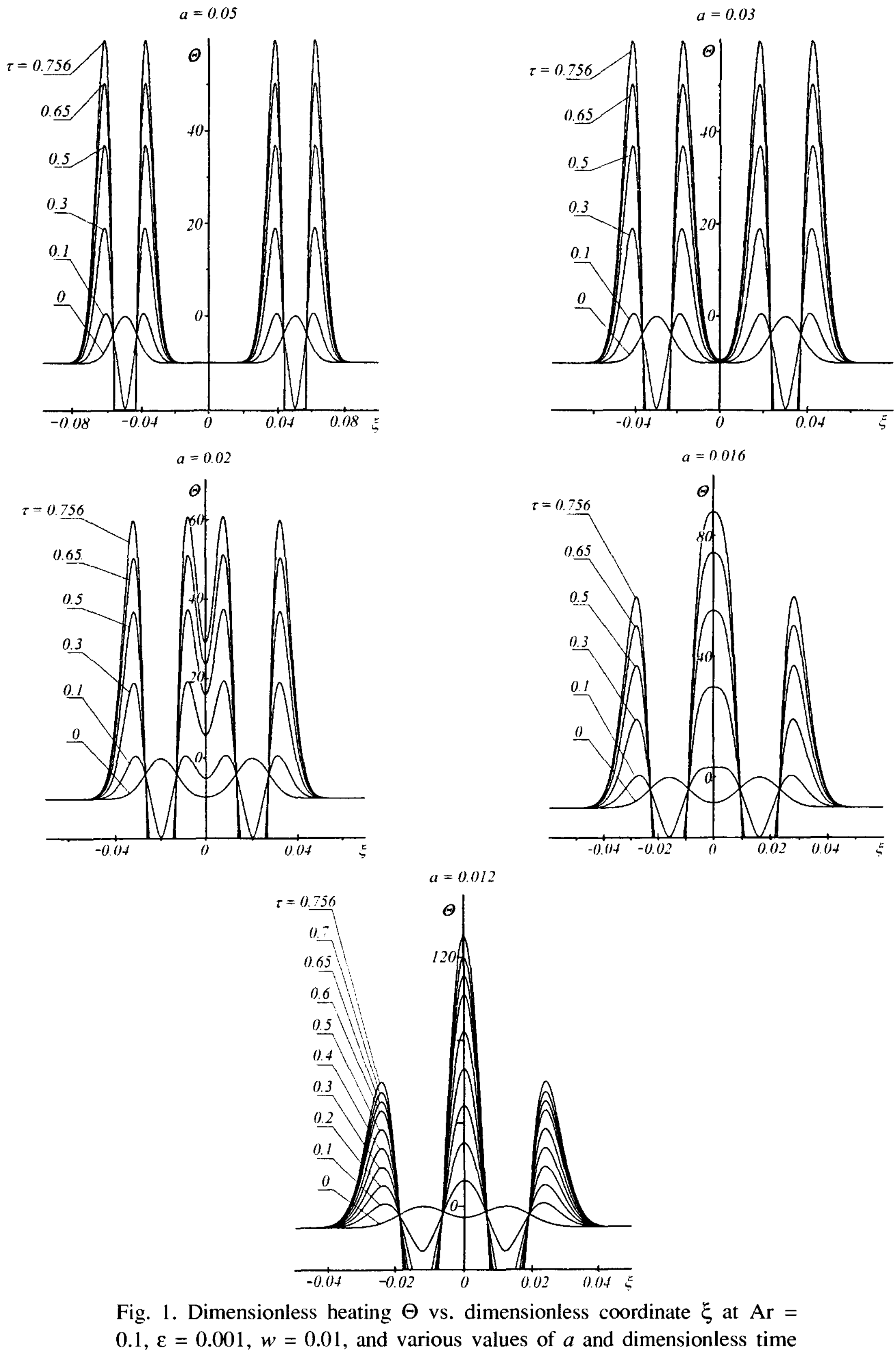

Fig. 1. Dimensionless heating Θ vs. dimensionless coordinate ξ at Ar = 0.1, ε = 0.001, w = 0.01, and various values of a and dimensionless time τ.

TABLE 1. Maxima of the Dimensionless Heating Θ as a Functions of the Dimensionless Time τ at Ar = 0.1, ε = 0.001, w = 0.01, and Various a

a	τ					
	0	0.1	0.3	0.5	0.65	0.756
0.05	0	0.605	19.111	36.937	50.330	59.800
0.03	0	0.605	19.111	36.937	50.330	59.800
0.02	0	0.674	19.453	37.544	51.137	60.747
0.016	0	3.012	29.766	55.249	74.362	87.870
0.012	0.032	12.563	48.267	84.045	110.928	129.95

$$\Theta^0(\xi) = \Theta(\xi, \tau)\Big|_{\tau=0} = \frac{[T_{in}(\xi, r) - T_0]E}{RT_0^2}, \quad (1)$$

where $T_{in}(x) = \exp\left\{-\left(\frac{x-a}{w}\right)^2\right\} + \exp\left\{-\left(\frac{x+a}{w}\right)^2\right\}$; a is a parameter assigning the distance between the "hot spots," and w is a factor modeling their width.

Results of the conducted quantitative parametric analysis of the mutual effect of two "hot spots" are given in Fig. 1 and Table 1. Calculations were made at the following values: T_0 = 1, w = 0.01, Ar = 0.1, and ε = 0.001. Only the parameter a changed successively (see equality (1)). At a = 0.05 and a = 0.03, each of the "hot spots" evolves in time independent of each other; in this case, each of the two initial Gaussian distributions is transformed to two "short-lived thermal waves" moving along the 0ξ axis to +∞ and −∞ in a time from τ = 0 to τ = 0.756.

At a = 0.02, the two "short-lived thermal waves," which move in opposition, are already mutually imposed, but, since this distance is not small enough, the effect of an increase in the total amplitude Θ(ξ, τ) is not observed in the figure. At a = 0.016, the two "short-lived thermal waves," which move in opposition, interact, while superimposing, so that, finally, the maximum of Θ(ξ, τ) increases to Θ ≅ 90 (τ = 0.756 at ξ = 0), whereas the "short-lived thermal waves," which move in opposite directions, have the maximum Θ ≅ 60 when τ = 0.756 (see the table), i.e., the same as with a = 0.05 and a = 0.03. The highest effect of intensification of "short-lived thermal waves" is attained at a = 0.012. Curiously, the resultant amplitude of Θ_{max} ≅ 130 is somewhat higher than the sum of the maxima Θ ≅ 60 of those "short-lived thermal waves" which move in opposite directions. The authors are apt to see in this fact the effect of "semilinearity" of the differential equation (1) from [5]. "Semilinearity" means the presence of a nonlinear Arrhenius-type heat source in it, with other terms of this equation being linear. The process of intensification of "short-lived thermal waves" is illustrated by the data, given in Table 1, in more detail. In closing we note that the effect of intensification of "hot spots" is observed at other values of the parameters Ar, w, a, and ε, with a qualitative picture of this intensification being similar to the described one.

NOTATION

$\Theta = (T - T_0)E/RT_0^2$, heating of the substance; $\xi = x/r$, spatial coordinate; $\tau = t/t^*$, time; r and t^*, space and time scale, respectively; $t^* = (c\rho RT_0^2)/(QEk(T_0))$, adiabatic scale of time; T_0, ambient temperature; R, gas constant; E, activation energy; c, heat capacity; ρ, density; Ar = RT_0/E, Arrhenius number; $T = T(x, t)$, temperature in the reaction zone; Fk = $QEr^2k(T_0)/(\lambda RT_0^2)$, Frank-Kamenetskii criterion; Q, thermal effect of the reaction (per unit volume); $T_{in}(x)$, distribution of temperature at the initial instant.

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